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The NMC 8-layer Global and Hemispheric Primitive Equation Models (8L-GLOPEP & 8L-HEMPEP) on a Longitude-Latitude (λ-ϕ) Grid

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THE NMC 8-LAYER GLOBAL AND HEMISPHERIC PRIMITIVE EQUATION

MODELS (8L-GLOPEP & 8L-HEMPEP)

ON A LONGITUDE-LATITUDE (λ-ϕ) GRID

This essay describes, in outline, the basic physics, structure, and design of the new NMC forecast models 8L-GLOPEP and 8L-HEMPEP. Where appropriate, differences between the 8-layer and 6-layer (6L PE) models are indicated.

I. Basic Equations

The fundamental physics of the forecast model, the hydrostatic (primitive) equations of motion derived from Newton's laws of motion, remain the same. The quantities to be forecast are the wind components u and v, potential temperature θ, "pressure thickness" (the difference in pressure between two arbitrary "σ" surfaces--see definitions below) p_σ, and a measure of moisture--here we shall use q the specific humidity (the 6L PE used precipitable water W--it makes no difference).

Each of these five quantities are assumed to describe the average conditions in eight suitably defined layers of the model atmosphere.

The equations are written in spherical coordinates (the 6L PE used Cartesian) which thus introduces some new terms into the equations and slightly changes the meaning of u and v. If we write λ for longitude and define it as increasing to the <u>east</u> (locations will be referenced as <u>east</u> of the Greenwich meridian, in the model) and ϕ the latitude, increasing northward, r is the radius of the (spherical) earth, then

$$u \equiv r \cos \phi \ \frac{d\lambda}{dt}$$

$$v \equiv r \frac{d\phi}{dt}$$

i.e. u is the west wind (when positive) and v is the south wind (when positive).

The equations are also written in terms of an arbitrary vertical coordinate σ. One may speak of surfaces of constant σ (just like one speaks of surfaces of constant pressure or height) along which other meteorological quantities can and usually do vary. The precise definition of σ is found below.

The wind component equations are

$$\frac{\partial u}{\partial t} + \dot{\sigma}\frac{\partial u}{\partial \sigma} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{uv}{r}\tan\phi$$
$$- fv + g\frac{\partial z}{\partial x} + c_p\theta\frac{\partial \pi}{\partial x} + F_x = 0 \qquad (1)$$

and

$$\frac{\partial v}{\partial t} + \dot{\sigma}\frac{\partial v}{\partial \sigma} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{u^2}{r}\tan\phi$$
$$+ fv + g\frac{\partial z}{\partial y} + c_p\theta\frac{\partial \pi}{\partial y} + F_y = 0 \qquad (2)$$

In these equations, $\dot{\sigma}$ is the vertical velocity and the terms involving it are the measures of vertical advection; f is the Coriolis parameter $2\Omega\sin\phi$ (Ω = earth's rotation rate) accounting for the rotation of our coordinate system; c_p is the specific heat of air at constant pressure; $\pi = (p/1000)^{R/c_p}$ (p in mb, R the universal gas constant) is known as the Exner function and is no more than a convenient measure of pressure; g is gravitational acceleration and z is the height above mean sea level of the sigma layer in which the variables are being forecast. In terms of the actual longitude latitude coordinates

$$\frac{\partial}{\partial x} = \frac{\partial}{r\cos\phi\,\partial\lambda} \qquad \frac{\partial}{\partial y} = \frac{\partial}{r\partial\phi}$$

i.e., the east-west or north-south increment of distance on the earth for a given increment of longitude or latitude. These derivatives are, of course, taken along surfaces of constant σ (that's the meaning of the partial derivative notation).

The first lines of equations (1) and (2) are the advective and metric terms, the second lines are the forces involved: Coriolis, height and pressure gradient, and friction, detailed below.

The temperature and specific humidity tendency equations are

$$\frac{\partial \theta}{\partial t} + \dot{\sigma}\frac{\partial \theta}{\partial \sigma} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + HC = 0 \qquad (3)$$

and

$$\frac{\partial q}{\partial t} + \dot{\sigma}\frac{\partial q}{\partial \sigma} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + EP = 0 \qquad (4)$$

both very similar in structure involving only advection (three dimensional) and either heating-and-cooling (HC) terms or evaporation-and-precipitation (EP) terms. These latter are described more fully below.

The pressure tendency equation (alias the equation of continuity, alias the conservation of mass) is

$$\frac{\partial p_\sigma}{\partial t} + \frac{\partial}{\partial \sigma}(\dot{\sigma}p_\sigma) + \frac{\partial}{\partial x}(up_\sigma) + \frac{\partial}{\partial y}(vp_\sigma) - \frac{p_\sigma v}{r}\tan\phi = 0 \qquad (5)$$

with the form of three-dimensional mass divergence plus a metric term. The value of p_σ is a measure of the mass between the σ surfaces involved.

Finally, the hydrostatic equation is

$$g\frac{\partial z}{\partial \sigma} + c_p\theta\frac{\partial \pi}{\partial \sigma} = 0 \qquad (6)$$

expressing the balance of vertical forces.

II. Vertical Structure

The definition of the vertical σ coordinate and the vertical 8-layer structure of the model go hand-in-hand. Consider any two surfaces in the atmosphere where the pressure is known for all x and y (or λ and ϕ) for example the surface of the earth and the tropopause. The σ coordinate then is defined as the fractional distance (with "distance" in pressure units) from the upper pressure surface p_U to the lower surface p_L thus:

$$\sigma = \frac{p - p_U}{p_L - p_U} \tag{7}$$

σ itself is nondimensional--it has the value 1 at $p = p_L$ and the value 0 at $p = p_U$. If we, for example, set $\sigma = \frac{1}{2}$ this defines a σ surface that exists halfway between the tropopause and the surface of the ground and therefore shares in the variations of both height and pressure of both of those defining surfaces.

Note that p_σ ($= \partial p/\partial\sigma$) just equals $p_L - p_U$ as a consequence of (7). Thus p_σ is quite properly spoken of as the "pressure thickness" and this pressure thickness is just the quantity that is forecast by equation (5).

The 8L GLOPEP and 8L HEMPEP models then use the definition of σ from (7) to specify just where the eight layers are. Two p_L/p_U pairs are used: one defines the tropospheric σ domain in which p_U is the tropopause pressure p** and p_L is the surface pressure p*; the other pair defines the stratospheric σ domain wherein p_U = 50 mb everywhere initially (a constant pressure as well as constant σ surface) and p_L = p**. Referencing Fig. 1, the tropospheric domain is divided into six layers by defining σ surfaces to exist at σ_T= 1/6, 2/6,...,5/6 and the stratosphere is divided into two

σ layers by a σ surface at $\sigma_S = 0.5$ making a total of eight layers in which our forecast variables are specified. The figure shows (and indeed the model has) a ninth layer, a "thetasphere", of constant potential temperature at the top. This layer serves as a computational upper boundary layer, necessary for the numerical good behavior of the model but not contributing to the meteorological forecasts. As far as meteorology goes, we have an 8-layer model. The figure also indicates which variables are actually forecast in the layers--note that the moisture, q, is carried only in the five lowest layers, elsewhere the model is dry--and the location of the diagnostic vertical velocity $\dot{\sigma}$. Physical boundary conditions are put on the model by requiring $\dot{\sigma} = 0$ at levels 8, 2, 0, and -1. Physically, this is stating that no mass (air) can pass into the ground, thru the tropopause, or into or out of the stratosphere and thetasphere.

(Earlier versions of the 8-layer models incorporated a surface boundary layer of fixed pressure thickness, p_σ, as the 6-layer model has--this was discarded in favor of increasing the resolution throughout the troposphere with six equal tropospheric layers. The bottom layer still serves as a "boundary layer" as far as surface effects are concerned. See below.)

The particular vertical structure shown here is not to be considered absolutely unchanging. Experience and experimentation may dictate changes in the layering structure, for example a layer might be moved from the troposphere to the stratosphere or the tropospheric layers might be made unequal in thickness thus increasing resolution by making thinner layers near critical regions such as the ground or tropopause. Such alterations will be documented, of course.

III. Horizontal Structure

In the horizontal plane, the data are carried at grid points defined at the intersections of meridians and latitude circles extending either over the whole globe for 8L GLOPEP or the Northern Hemisphere for 8L HEMPEP. Other than the extent of the grid, there are no differences between the global and hemispheric models. The equatorial boundary is taken care of by assuming symmetry across the equator for 8L HEMPEP.

The mesh size is easily variable and depends basically on the ability of the available computer to get out a timely forecast. At the time of writing (November 1973), the plans call for a 2.5° λ-ϕ mesh in the global version used as a FINAL forecast run and a 2° λ-ϕ mesh in the Northern Hemisphere version as the OPERATIONAL model. Further reductions of the grid size are contemplated both to press the machine to its limit and to study the mesh size effects upon the forecast quality.

IV. Initial Conditions

The end results of the Hough function analysis are heights, winds, and temperatures on the mandatory levels up to at least 50 mb (and higher if adequate data becomes consistently available) plus surface temperature, tropopause pressure, and relative humidity on mandatory levels up to and including 300 mb. Transferring this information to the eight layers of the model is done in essentially the same manner as the 6L PE model (see Shuman and Hovermale, J. Appl. Meteor., V7, p. 525, 1968). There are some small differences: the temperature in the lowest layer is determined by interpolation between the surface temperature and the 850 mb temperature (provided the surface elevation is below the 850 mb surface); the bottom of the thetasphere, level 0, is given an initial pressure of 50 mb; the relative humidity is interpolated (linearly with the natural log of pressure) to the centers of the σ layers and there converted to a mean specific humidity for the layer. The winds are also interpolated in the same manner but, unlike the 6L PE model (at the present time anyway), they are not rendered nondivergent nor do they have the 12-hour forecast divergence added to them.

There are difficulties in the initial conditions that make themselves evident in the first half day or so of the forecast--methods of initializing the forecasts, without degrading their quality, are under study and will be reported upon in due course.

V. Additional Physical Effects

A. Friction

The model senses the ground by incorporation of a frictional drag term--there is no other friction in the model, only a surface friction term. The expression is based on the Prandtl layer theory and takes the form

$$\left.\begin{aligned} F_x &= \frac{g\rho}{\Delta p} C_D \left| \vec{V} \right| u \\ F_y &= \frac{g\rho}{\Delta p} C_D \left| \vec{V} \right| v \end{aligned}\right\} \qquad (8)$$

where Δp and ρ are the pressure thickness and mean density respectively of the lowest model layer, $\vec{V}$ the vector wind (with u and v the components of course) in that layer and C_D the drag coefficient. The drag coefficient varies with the nature of the terrain and an explanation of how it is evaluated can be found in Cressman's article, Monthly Weather Review, 1960. The 8L models are using the same C_D as the 6L PE does except of course on a λ-ϕ grid and with a new set of values derived for the Southern Hemisphere.

B. Short Wave (Solar) Radiation

At any moment during the forecast we know, by calculation, what the solar zenith angle, ξ, is at each grid point. This knowledge plus a simple calculation of the precipitable water W

$$W = \frac{\Delta p}{g} q \tag{9}$$

for the layer with pressure thickness Δp and forecast specific humidity q enables us to compute what the radiation people call the path length

$$u = W \sec \xi \tag{10}$$

which represents the "amount", in some convenient units, of water vapor that the beam of solar radiation passes through and is at the same time partially absorbed by. Published computations exist (Manabe and Wetherald, J. Atmos. Sci., Vol. 24, p. 241, 1967) that relate path length to the actual quantity of energy absorbed from a beam so it is a straightforward task to follow a sunbeam down through the lowest five layers of the model (the only ones that contain water) and calculate how much energy is absorbed and thus how much the layer is heated. This is part of the HC term of equation (3).

All of the above applies to clear sky conditions. If the descending beam hits a cloudy layer (which for present purposes is defined to be a layer with 90 percent humidity or greater), two things happen: 1) a fair portion of the beam is reflected back upward from the top of the layer thus causing less radiation to be available for subsequent absorption below (cloudy days are cooler days) and 2) both the reflected and transmitted radiation become diffuse rather than beamish and equation (10) no longer applies directly. No problem. Instead we use

$$u = 1.66\ W \tag{11}$$

where 1.66 is an equivalency sort of a term, known as the Elsasser Diffusion factor, enabling us to use beam absorption calculations even though the radiation is really diffuse. The now depleted and diffuse radiation proceeds through the cloud layer and any clear layers below being further absorbed as it goes. Any further encounters with clouds will cause additional upward reflection but no additional diffusion. Once diffuse always diffuse. When the radiation reaches the ground, it undergoes a final reflection and we proceed to calculate the further absorptions of the upwelling radiation streams, radiation reflected both from the ground and any cloud layers met on the way down.

The albedos (the ratio of reflected to incident radiation) for the possibly cloudy layers from the ground up are, at present, 0.7, 0.6, 0.5, 0.4, and 0.3; that is, low clouds reflect a greater portion of what hits them than high clouds. The albedo of the ground is a geographic variable. All sorts of thermal things happen at the ground besides reflection of solar radiation--these are detailed below.

The 6L PE model incorporated a portion of these shortwave calculations--in effect the heating from the downward coursing stream was included but not the upward stream heating. It's a small refinement but one that was easy to incorporate.

C. Long Wave (Terrestrial) Radiation

Calculation of the heating and cooling due to long-wave radiation is somewhat more involved (and for that reason was not included in the 6L PE model) in that the radiant energy is both emitted and absorbed *in situ* by the water vapor of the model atmosphere itself.

The computational procedure is one of determining the radiative flux at each level (interfaces between layers) by adding up the radiation emitted from each adjacent layer, plus the radiation emitted from the layers once removed less the portion of that radiation absorbed in the adjacent layers, plus the radiation emitted from layers twice removed less the absorption in the two intervening layers and so on and on. The procedure does not reach out forever, of course, it will terminate at the top of the moisture containing portion of the model (above layer five) or at the edge of a cloudy layer (90 percent humidity or more again) or at the ground. A cloud layer is considered a "black body" (in radiation terms)--all radiation hitting it is completely absorbed, none passes through, and its radiative emission depends only upon its temperature. Thus, the summation described above stops at a cloud top or bottom. The ground is also a black body. Once all the fluxes, both upward and downward at all levels, are computed it is a simple task to ascertain the net flux in each layer which is the net energy gain or loss of the layer and thus the temperature change, another contribution to the HC term of equation (3).

For those with a particular interest in the matter, the flux calculations are made by integrating the wavelength integrated emissivity gradients times the black body emission over path length. We are using emissivities derived by P. M. Kuhn, J. Appl. Meteor., V. 2, pa 368, 1963.

D. Surface Energy Exchanges and Temperatures

We are introducing some new physical effects to the 8L models to describe, hopefully, the various energy exchanges at the ground (and ocean) surface with the end result of being able to make a forecast of surface temperature. These effects were present in the 6L PE only in a very emasculated form and were not capable of producing surface temperatures.

The method and underlying physical assumption is to compute all the various forms of energy flux through the surface of the ground, in either direction, and require that the resultant flux, the net flux, shall be zero. The various surface energy flux terms and formulae for computation are:

1. Net short wave radiation. This is a portion of the calculation made in section V. B, and is simply the downward flowing radiation less however much is reflected at the ground.

2. Downward streaming long wave radiation. This, similarly, is a portion of the calculations from section V. C, found by the summation procedure described there.

3. Sensible heat flux. The expression is

$$H = \rho \left| \vec{V} \right| C_D \, c_p \, (\theta - \theta^*) \tag{12}$$

The density ρ, wind $\vec{V}$, and potential temperature θ are those of the lowest model layer, θ^* is the ground or sea surface temperature. This expression allows for heat flux in either direction and over land or water thus incorporating heating or cooling of air over land and cooling over water, three effects not included in the 6L PE model. Note that we are using equation (12) in two capacities. It is both part of the surface flux calculation (our present concern) and the heat energy passing into or out of the lowest layer of the model contributes to temperature changes in that layer--a further portion of the HC term of equation (3). A special note: if H is positive, a statically stable situation, its value (and quantities associated with it) is reduced by a factor of 10.

4. Latent heat flux. The expression is

$$LH = \rho \left| \vec{V} \right| C_D \, L \, (q - w \, q^*_s) \tag{13}$$

which is obviously similar to the sensible flux expression, except that here we have the latent heat of evaporation L and the specific humidity. q^*_s is the saturation specific humidity at the surface, a function of the surface temperature and pressure only, and w is the "potential evaporation," a sort of ground wetness parameter described by Saltzman in Tellus, v. 19, p. 219, in 1967. Over the oceans $w = 1$, over land w will be less than one (but always greater than zero) expressing the physical effect of land surfaces having a less than complete availability of water for evaporation purposes. In the present model, we are setting w equal to one minus the albedo, i.e., assuming that low albedo areas (forest lands) are relatively moist and conversely (i.e., deserts are dry). As with equation (12), the latent flux can go either away from the surface (evaporation) or toward the surface (dew formation). Equation (13) is, like (12), also used in a double capacity both for the flux determination and, with the latent energy flux converted to amount of moisture units, as a contribution to the EP term of equation (4) for the lowest layer moisture tendency. Thus the 8L models include evaporation over land and water surfaces (and dew formation, too) while the 6L PE model included only evaporation over water as a source (or sink) of moisture for the model atmosphere.

5. Energy flux into or out of the ground. Some sort of explicit ground storage source/sink term could be incorporated into the model if we wanted to go into the geology business. For the time being we don't and instead we will assert that the effect of the ground storage will be such as to ameliorate any changes in the surface temperature computed without reference to the ground flux term. The ground term will by implication always reduce surface temperature changes, always be a brake.

6. Upward streaming Long Wave Radiation. This is really the leftovers term, the term that is used to balance out the other four flux terms to get a net flux of zero. The black body radiation from the ground is

$$LW_g = \sigma(T_E^*)^4 \tag{14}$$

σ is the Stephan-Boltzman constant, T_E^* the Kelvin scale surface temperature. The fluxes of paragraphs 1-4 are summed, set equal to LW_g and equation (14) is solved for T_E^* to give the equilibrium surface temperature. The ground storage effect is then introduced by computing a tendency for the surface temperature δT^* by

$$\delta T^* = \beta(T^*_E - T^*) \tag{15}$$

where T^* is the surface temperature from the previous time step of the forecast and $\beta(= 0.05)$ specifies the amount of lag, "caused" by the "ground storage", in the surface temperature change. Over open water $\beta = 0$, i.e., the sea temperature remains constant throughout the forecast.

VI. Precipitation Forecasts

The quantitative precipitation forecast is simply derived from the q forecasts, equation (4), by inquiring whether the forecast value of q is greater than q_s the saturation specific humidity of each layer. If it is not ($q \leq q_s$), nothing further is done and the forecast proceeds to the next time. If supersaturation does exist ($q \geq q_s$), then the excess moisture is assumed to be condensed as rain and falls from the layer. At the same time q is reduced to q_s (the supersaturation is not allowed to continue unless it is forecast to recur in the next step), a contribution to the EP term of (4), and a latent heat contribution is added to the HC term of the temperature forecast equation for the layer. The amount of latent heat and consequent temperature change depends of course upon the amount of water condensed.

This same procedure repeats for each of the five moisture-bearing layers of the model.

The rain does not fall from each layer undisturbed. If it falls into an unsaturated layer, sufficient rain will evaporate to bring the layer to saturation. Any rain left will continue to the next layer below where the same process repeats. Any evaporation contributes to the EP and HC terms of the q and θ equations for the layers.

The 6L PE model has an additional precipitation forecast--the so-called convective precipitation section. At present, we are not planning to include this in the 8L models in the same form as it is in the 6L PE. Instead our intent is to introduce a more general parameterization of cumulus scale convection which will include, to be sure, convective precipitation as one of its by-products but also include vertical heat and momentum transports of the sort that seem to be of considerable importance in the Tropics. After all, there's a lot of Tropics in a global forecast model. The form of the cumulus parameterization is still under development and will be reported upon in due course.

VII. Convective Adjustments

When a super-adiabatic lapse rate develops, both the atmosphere and the numerical models become unstable. The atmosphere resolves the instability by mixing in the vertical all by itself, so to speak. We have to tell the model what to do and naturally we follow the atmosphere's lead and induce vertical mixing between the unstable layers.

The procedure is quite straightforward: if two dry layers are found to be unstable, they are both reassigned a mass weighted mean potential temperature

$$\theta_m = \frac{\theta_U \Delta p_U + \theta_L \Delta p_L}{\Delta p_U + \Delta p_L}$$

Where U and L refer to the upper and lower layers respectively. The test then repeats for the next pair of layers above using the newly adjusted temperature in what is now the lower of the two layers.

If a layer is saturated, it is tested for stability with respect to the layer above it using a saturation pseudo-adiabat and if unstable the two layers have their temperatures adjusted to lie on the appropriate pseudo-adiabatic temperature curve.

VIII. Output of Forecasts

After marching forward in time for many 10-minute time steps, the model atmosphere eventually reaches an appropriate hour for output. The forecasts that go out from NMC are for the most part not in σ coordinates but generally on constant pressure or constant height surfaces. (The few exceptions are such items as boundary layer winds and temperatures--and hopefully surface temperatures if they turn out to be any good--and tropopause related quantities.) It is necessary to interpolate from the σ coordinate to the pressure coordinate. This is accomplished in the same way as the 6L PE model--basically either a hydrostatic calculation of heights at known mandatory pressure surfaces given the heights, pressure and temperatures at surrounding σ surfaces or a linear interpolation of $\vec{V}$, θ, or q (linear with π) from the σ layers to the pressure surfaces. The same special underground extrapolation to sea level (or 1000 mb) is carried in the 8-layer models as is found in the 6L PE model.

There is at the time of writing an investigation proceeding into the proper method of smoothing, for output purposes, the fields on a λ-ϕ grid. The question comes up as to what to do with the poleward meridian convergence, as the x coordinate scale changes markedly, while there is no corresponding change in the latitude scale, the y coordinate. This will be reported upon in due course.

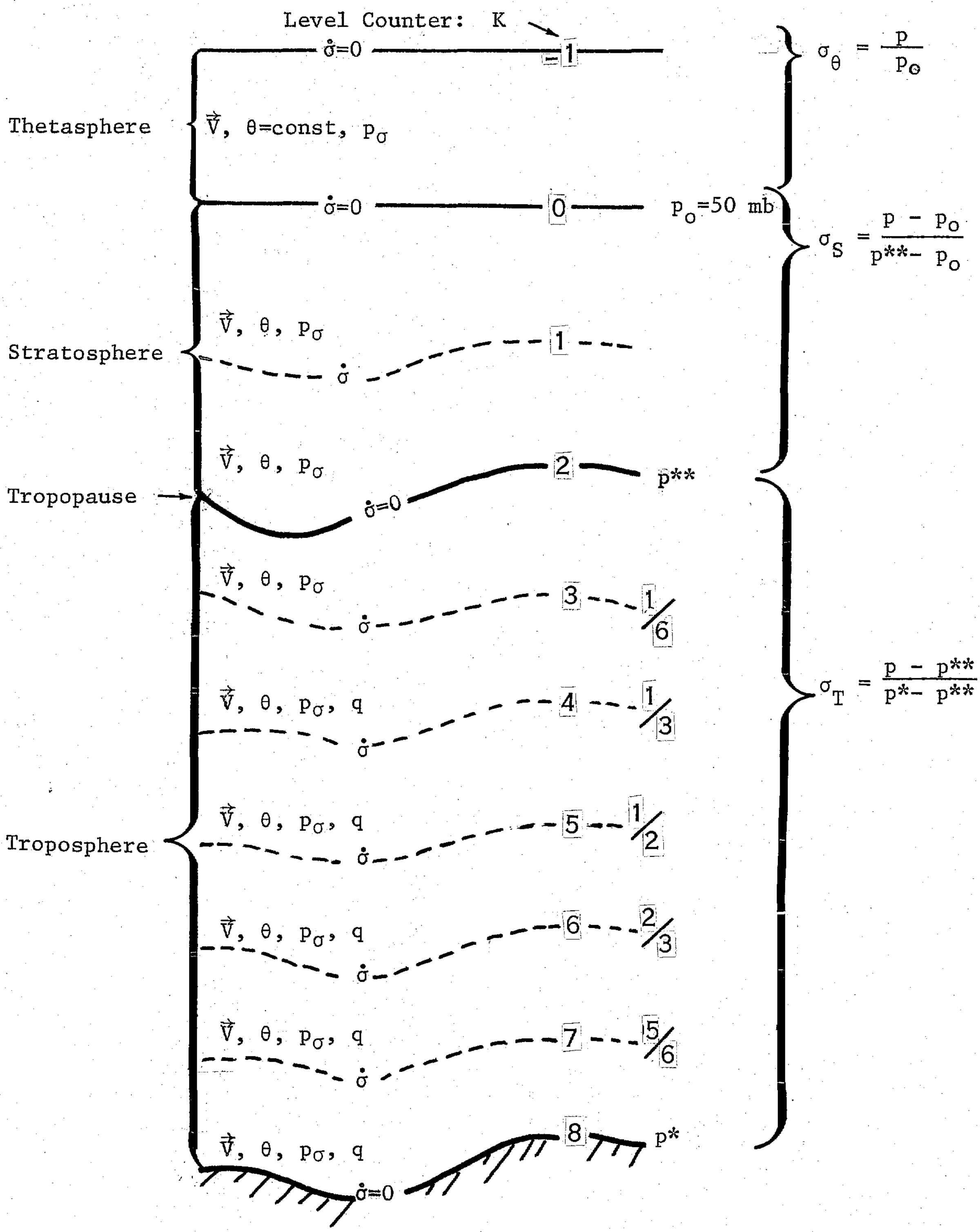

Fig. 1.

Vertical Structure of 8L GLOPEP and 8L HEMPEP